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THE IMPORTANCE OF PROVEN TREATMENT AS A VACCINE MODEL FOR COVID-19

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Abstract. The COVID-19 global pandemic has posed the biggest medical challenge of the past three years, and efforts to eradicate it have involved cross-border collaborations.

Recent studies claim that the development and distribution of vaccines is an effective approach to end this infectious disease. However, this study demonstrates that a vaccine alone may not be enough due to differences in the rates of waning immunity induced by infection and vaccination. We propose that both vaccination and treatment are required to halt COVID-19 disease transmission and end the pandemic. From an epidemiological perspective, we employ the SVEIRS model by introducing vaccination and treatment into the compartmental epidemic model SIR.

The experimental results suggest that the pandemic can be ended by performing both vaccination and treatment. However, the development and distribution of vaccines and treatments take time, and control measures must continue until the disease is completely eradicated.

Keywords: COVID-19, Mathematical Model, Vaccine, Treatment.

AMS Subject Classification: 34D, 34H, 90A, 92B.

1. Recent trends in COVID-19 control

Since the first reported outbreak of the coronavirus disease 2019 (COVID-19) in Wuhan, China in late 2019, the COVID-19 pandemic has been the most formidable medical challenge for humankind. We are witnessing global efforts towards the eradication of COVID-19, as this infectious disease has proved to be a problem that cannot be solved only on a national scale. Numerous studies have been conducted to analyze and predict the spread of infection based on mathematical transmission dynamics models of infectious diseases [3, 4, 8, 14, 30, 34], and many countermeasures and epidemic prevention methods have been proposed [5, 6, 11–13, 17–19, 21, 22, 25, 31, 32].

According to recent research [16, 35] and references therein, it is widely accepted that the world will return to its prepandemic normalcy after safe and effective vaccines become available

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Data Availability: The data used to generate Figure 2 is available for download on GitHub at the following link: https://github.com/owid/covid-19-datahttps://github.com/owid/covid-19-data.
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and a global vaccine program is successfully implemented. Therefore, numerous pharmaceutical companies are focusing their efforts on vaccine development, and each country is continuing to implement policies to suppress the spread of infection while waiting for the release of vaccines that have passed clinical trials. Vaccination offers a powerful tool for disease control by protecting susceptible individuals against infection [24]. Generally, however, providing vaccinations to the entire population simultaneously is not feasible, and only a limited portion of the population can be vaccinated at a time. Furthermore, vaccines occasionally provide limited immunity due to the natural waning of immunity in the host or antigenic variations in the pathogen [2, 32]. Thus, the assumption that vaccine administration will end this devastating infectious disease is certain only when the waning rate of vaccine-induced immunity is considered to be equal to or less than that of immunity acquired by infection in compartmental epidemic models, as shown in Fig.1. However, significant evidence has indicated that the two waning rates of infection- and vaccine-induced immunity are different [9, 29], which would lead to a failure in the control of this infectious disease.

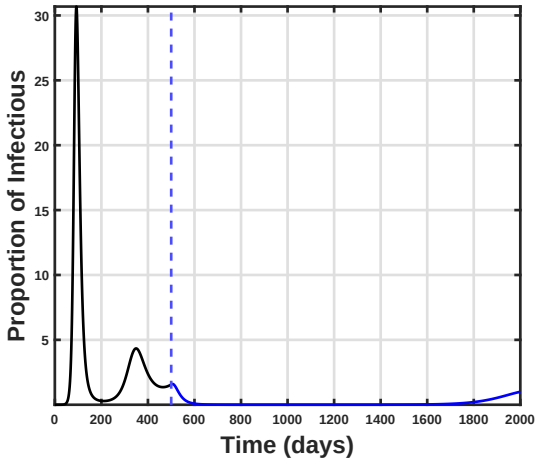


FIGURE 1. SVEIRS model (1) with equal waning vaccine and infection-induced immunity rates ($\alpha = \theta$) and no treatment ($\kappa = 0$) shows a surge after 1800 days. In this simulation, we set the vaccine to be distributed at $t = 500$.

Taking these considerations into account, we suggest eradicating the disease by focusing on the impact of treatment on the mechanism of the dynamic transmission of the disease. The proposed compartmental epidemic model indicates that it is difficult to end the disease with a vaccine alone, whereas it becomes possible with accompanying treatment.

The aim of this study is to advise researchers on the mathematical modeling of epidemics and provide public health policy makers with a novel perspective in disease control by considering the impact of treatment as well as vaccination. The results obtained in this study provide important insights into the development of an effective vaccine and therapeutic strategies for controlling the spread of infectious diseases.

2. SVEIRS MODEL FORMULATION

Mathematical modeling has been used to predict the transmission dynamics of infectious disease and evaluate the effects of a number of interventions, such as vaccination and treatment. We search for an approach to end infectious diseases through compartmental epidemic models. Because an essential factor in ending infectious diseases is the development of vaccines and therapeutics, we address the effect on the control of the spread of infectious diseases when vaccines and treatments are applied side-by-side to an epidemic transmission model. We model the transmission dynamics of infectious diseases into the compartment epidemic scenario.

The backbone frame of most epidemiological models originates from the classical compartmental model of Kermack and McKendrick [20], as it assumes that susceptible individuals become infected upon contact with infectious individuals and subsequently recover with acquired immunity. As a typical approach in mathematical epidemiology, we divide a population into three compartments: susceptible, infectious, and recovered. In this model, we consider a latent period between being infected and becoming infectious. We will develop a mathematical model for COVID-19 by adding the latent stage E compartment to the above SIR model. In addition, we will add the V compartment to our model to evaluate the effect of vaccines. Additionally, significant evidence is showing that the immunity of COVID-19 does not persist permanently, but wanes and disappears [28]. If the immunity is completely lost, an individual moves from R to S or from V to S. We apply vaccines and treatments sequentially at a time interval after the outbreak of infectious diseases. This work advances the research on vaccination strategies and epidemic modeling approaches introduced by [7,37], by considering combined interventions that incorporate both vaccination and treatment.

The model is described as in Fig.2.

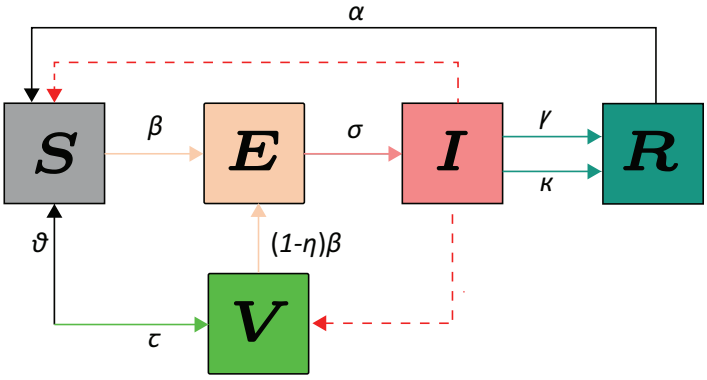


FIGURE 2. SVEIRS compartment model.

Under the above conditions, the total population N is divided into five compartments. Let $S(t), V(t), E(t), I(t)$, and $R(t)$ be the number of susceptible, vaccinated, exposed, infectious, and recovered individuals at time t, respectively. As shown in Fig.2, we formulate the dynamics of transmission into the following system of differential equations:

$$\begin{cases} \frac{dS}{dt} = -\beta\frac{I}{N}S - \tau S + \alpha R + \theta V \\ \frac{dV}{dt} = \tau S - (1-\eta)\beta\frac{I}{N}V - \theta V \\ \frac{dE}{dt} = \beta\frac{I}{N}S + (1-\eta)\beta\frac{I}{N}V - \sigma E \\ \frac{dI}{dt} = \sigma E - (\gamma+\kappa)I \\ \frac{dR}{dt} = (\gamma+\kappa k)I - \alpha R, \end{cases} \tag{1}$$

where $N = S(t) + V(t) + E(t) + I(t) + R(t)$ is time independent since $\frac{dN}{dt} = 0$. Here, parameter β denotes the transmission coefficient, σ is the incubation rate, γ is the recovery rate, τ is the vaccination rate, η is the vaccine efficacy, and κ is the treatment efficacy. We take waning immunity into account, assuming that the immunity by natural infection or vaccination wanes over time. By α and θ, we denote the waning rates of natural infection- and vaccine-induced immunity, respectively. In general, however, natural infection-induced antibody titers are higher than the vaccine-induced antibody titers [9, 29].

TABLE 1. Parameter Estimates for SEIR and SIR Models.

Country	**Model**	β	R_0	σ	γ	E_0
Japan	SEIR	0.1532	1.532	0.2	0.1	160
	SIR	0.1349	1.349	-	0.1	-
UK	SEIR	0.5588	5.588	0.2	0.1	351
	SIR	0.2982	2.982	-	0.1	-
US	SEIR	0.7920	7.920	0.2	0.1	2130
	SIR	0.3771	3.771	-	0.1	-
Spain	SEIR	0.7299	7.299	0.2	0.1	3464
	SIR	0.3721	3.721	-	0.1	-
South Korea	SEIR	0.2619	2.619	0.2046	0.1	2040
	SIR	0.3051	3.051	-	0.1	-

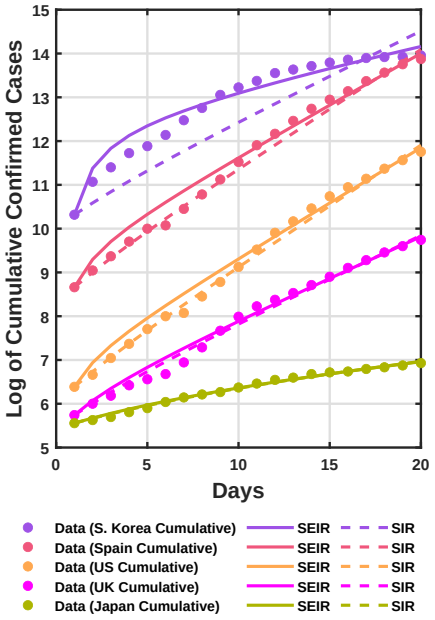


FIGURE 3. Model fits to early COVID-19 data for different regions. To ensure visual clarity and prevent any overlapping, the data for South Korea, Spain, and the UK were adjusted with offsets of 5, 2, and 0.5, respectively. The normalized infection data underwent a thresholding process to guarantee consistent initial conditions among countries. The first 20 days of the epidemic were analyzed in relation to the infection rate exceeding the defined threshold of $\frac{\text{infection}}{\text{population}} \times 10^6 \geq 2$. We followed the methodology outlined in [3].

3. EXPERIMENTAL EVALUATION OF EFFECT OF VACCINATION AND TREATMENT

The following section focuses on the assessment of the efficiency derived from integrating vaccination and treatment strategies in mitigating the proliferation of COVID-19. In order to accomplish this, we employed the SEIR and SIR models for the estimation of crucial parameters including β (transmission rate), σ (incubation rate), γ (recovery rate), and E_0 (initial exposed population). The parameters were calculated using early pandemic data from Japan, the UK, the US, Spain, and South Korea. The estimates for both the SEIR and SIR models are summarized in Table 1 and illustrated in Fig.3. To achieve optimal results for these parameters, we employed the Poisson likelihood estimation method in combination with MATLAB's `fmincon` optimization

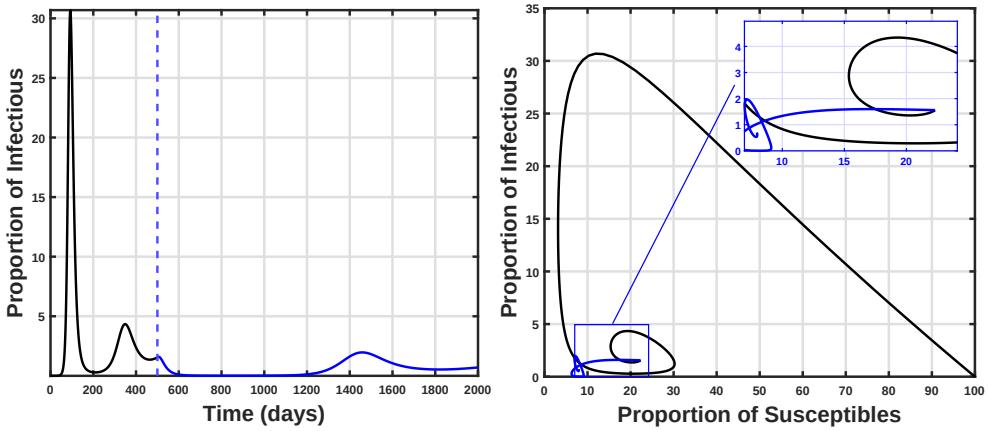
function. The rationale behind choosing this approach was its proven efficacy in handling count data, such as the daily number of infections, which typically exhibits a Poisson distribution. The Poisson likelihood function was used to quantify the disparity between the estimated number of new infections according to the model and the actual reported cases. To clarify further, our intention was to maximize the log-likelihood function, which can be defined as:

$$\text{log-likelihood} = \sum_{i=1}^{n} \left(y_i \log(\hat{y}_i) - \hat{y}_i \right),$$

where y_i represents the infectious cumulative data, and $\hat{y}_i$ is the model-prediction for day i. The `fmincon` function was used to minimize the negative log-likelihood and determine the best-fit estimates for several parameters in an epidemiological model. To ensure that the parameter values remained biologically feasible and consistent with known epidemiological ranges, constraints were applied. We conducted an experiment to evaluate the effect of the combination of vaccines and treatments on the control of the spread of infectious diseases. In particular, in the experiment, we reflect the differences between the waning of infection-induced immunity and vaccine-induced immunity [9, 29]. Since there are fewer antibodies produced by vaccines than by natural infections, represented as $\theta = m\alpha (m \geq 1)$. For the default experiment, we set the population size $N = 100,000$ and use the mean values of the estimated parameters (Table 2), $\beta = 0.4992$, $\sigma = 0.2$, $\gamma = 0.1$, and the initial proportion of active cases $I(0) = 0.05$ [3]. Research on the immune system of COVID-19 is still underway. Considering the fact that the natural immunity lasts for several months [15, 27], we first consider when the natural immunity of COVID-19 lasts for six months. Additionally, we assume that there is a twofold difference between antibodies produced by natural infections and antibodies produced by vaccines. We will experiment on various cases of the duration of the two types of immunity in sensitivity studies. Here, the above assumption implies that $\alpha = \frac{1}{365}$ and $\theta = 2\alpha$ [9, 29]. Then, we use the vaccine efficacy $\eta = 0.8$ [23,36] and set the vaccination rate to $\tau = 0.05$. We will also experiment using various vaccination rates in sensitivity studies. To investigate the effect of the vaccination and treatment, we assume that the vaccine was distributed 500 days after the outbreak of the infectious disease and that the drug treatment was developed 1000 days later. Finally, sensitivity experiments were conducted with different parameters in different environments to verify our claims.

3.1. **Effect of implementing only vaccination.** We first check for changes in the number of infections when only vaccines were provided. As depicted in Fig.4 (a), the experiment on the spread of infectious disease by the infectious population reaches its peak at $t = 94$ (day), where the populations are given as $S(94) = 11,841$, $E(94) = 14932$, $I(94) = 30,690$ (30.69%), and $R(94) = 42,538$.

When the vaccine is assumed to be distributed 500 days after the outbreak (the vaccination rate $\tau = 0.05$) at $t = 500$ (where $S(500) = 22,310$, $V(500) = 0$, $E(500) = 834$, $I(500) = 1,560$, and $R(500) = 75,296$), the transmission of infection exhibits a remarkable decrease again. However, it maintains a constant level after approximately 1000 days with $S(1000) = 8,757$, $V(1000) = 69,832$, $E(1000) = 6$, $I(1000) = 11$, and $R(1000) = 21,394$ without entering a risk-free state. From this point on, the infection appears to reach an epidemic state and persist in that state. The population at $t = 2000$ appears as $S = 7,947$, $V = 63,880$, $E = 359$, $I = 702$, and $R = 27,112$. In the phase plane shown in Fig.4 (b), when there is no vaccine, the endemic equilibrium is maintained around the point $(22.31, 1.56)$, which is the end point of the black spiral curve and is related to $(S(500), I(500))$. After the vaccination is conducted, the epidemic equilibrium exits the point $(22.31, 1.56)$ to $(8.757, 0.011)$ and stays around that point, which marks the epidemic equilibrium by vaccination. However, the production of infectious individuals still remains positive.



(a) Trajectory of epidemic transmission. (b) Phase plane of epidemic wave.

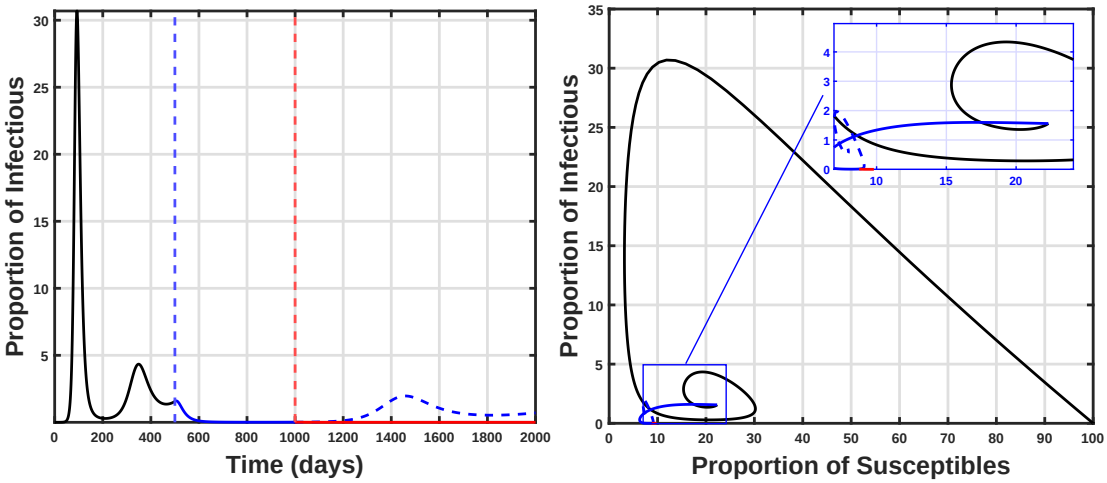
FIGURE 4. Simulated result of the effect of vaccination only, obtained by the SVEIRS model with parameters $\beta = 0.4992$, $\gamma = 0.1$, $\sigma = 0.2$, $I(0) = 0.05$, and $N = 100,000$. The different immunity waning rate parameters are $\alpha = \frac{1}{365}$, $\theta = 2\alpha$. The vaccination program is implemented 500 days after the outbreak of the disease (blue dashed vertical line in (a)). After application of the vaccine, the trajectory is represented by a blue curve. The vaccine-related parameters are $\tau = 0.05$, $\eta = 0.8$. (a) Trajectory of epidemic transmission up to 2000 days. (b) Phase plane with proportion of susceptible and infectious individuals.

This experiment implies that even when the vaccine is available, a many infected people are present in the public. This indicates an unpleasant cohabitation that can potentially cause a pandemic at any time since the infectious individuals are still present.

3.2. **Effect of implementing both vaccination and treatment.** Subsequently, we evaluated the effect of vaccination and treatment due to the proposed SVEIRS model. We assumed that the efficacy of treatment was $\kappa = 0.05$ and the treatment developed 1000 days after the outbreak of the infectious disease.

According to the experimental result shown in Fig.5, the development of therapeutic agents considerably helps control the spread of infectious diseases. As discussed in the previous section, the vaccination effectively controls over the spread of infectious diseases; however, limitations are encountered when attempting to completely eradicating the diseases. Thus, the spread of infectious diseases should be controlled via both treatment and vaccination to eradicate the diseases.

We analyze the experimental results to examine the effectiveness of the treatment in further detail. When a treatment is developed and provided at $t = 1000$, the spread of the infectious disease rapidly decreases, and the disease is completely eradicated after 1988 days. This indicates that treatment development leads to a disease-free equilibrium, whereas solely vaccination fails to eradicate the disease, and the epidemic state prolongs. Further, solely treatment development cannot immediately eradicate the disease owing to the low recovery efficacy from treatment ($\kappa = 0.05$), which is set equal to the vaccination efficacy τ in the experiment. Reports by [1,10,26] claim that a significant number of individuals are infected by asymptomatic patients. By varying this number, we can expect an improvement in the effect of treatment development. In Fig.5, we depict the results related to treatment in red and vaccination in blue. At $t = 2000$, the populations are estimated at $S = 9,804$, $V = 88,808$, $E = 0$, $I = 0$, and $R = 1,388$. After treatment development, the epidemic equilibrium moves from the point $(8.102, 0.560)$ to the disease-free equilibrium $(9.804, 0)$ and stays on the x axis, where $I = 0$. That is, both vaccination and treatment helped suppress the spread of the disease and positively eradicate it.



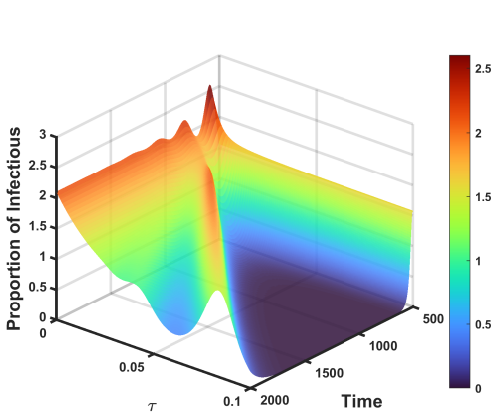
(a) Trajectory of epidemic transmission. (b) Phase plane of epidemic wave.

FIGURE 5. Simulated results of the effect of vaccination and treatment. $\beta = 0.4992$, $\gamma = 0.1$, $\sigma = 0.2$, $I(0) = 0.05$, $N = 100,000$, $\alpha = \frac{1}{365}$, and $\theta = 2\alpha$. The vaccination program is implemented 500 days after the outbreak of the disease (blue dashed vertical line), and the treatment is distributed 1000 days after the outbreak (red dashed vertical line in (a)). The trajectory of transmission observed after administering the vaccine is denoted by a blue solid curve; the trajectory observed after treatment is denoted by a red solid curve. The administration of the vaccine is denoted by a blue dotted line, for comparison. Vaccine-related parameters are $\tau = 0.05$ and $\eta = 0.8$, and treatment-related parameter is $\kappa = 0.05$. (a) Trajectory of the proportion of infections up to 2000 days, (b) Phase plane with the proportion of susceptible and infectious individuals, when treatment is provided after 1000 days.

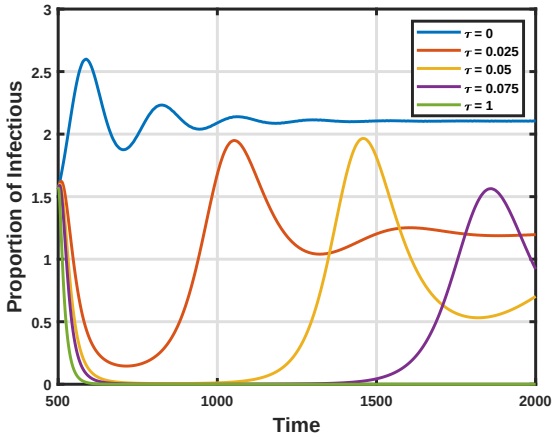
3.3. **Experimental sensitivity study on control effect by vaccine and treatment implementation.** Limited cases have been reported thus far in literature regarding the extent to which the loss of immunity owing to vaccines and natural healing is affected. Therefore, the spread of the disease when the values of the parameters are varied should be investigated. We experiment on the sensitivity of the control effect of the spread of infectious diseases according to vaccination and treatment by varying the parameters of the SVEIRS model. In the experiment, the default values are assigned as $\tau = \kappa = 0.05, \alpha = 1/365$, and $m(= \theta/\alpha) = 2$, and the values of the remaining parameters are assigned as listed in Table 1.

3.3.1. *Sensitivity study of the vaccination rate and treatment efficacy.* The number of the infectious individuals reaches a peak of 30.69% (30,690 of 100,000) on the 94th day after the onset. With no vaccination and treatment, the disease lingers indefinitely with an epidemic equilibrium 2.1% after some oscillation, as shown in Fig.6 (b) with $\tau = 0$. Introducing treatment ($\kappa = 0.05$) alongside vaccination leads to disease-free equilibrium. However, the timeline for attaining this goal is subject to the vaccination rate (Table 2). High vaccination rates, like $\tau = 0.1$, are challenging because of logistical and social constraints. Moderate vaccination rates (e.g., $\tau = 0.05$) combined with sustained treatment efforts are essential for disease control. While higher vaccination rates expedite eradication, realistic public health strategies should concentrate on attainable vaccination targets alongside treatment to control the epidemic effectively.

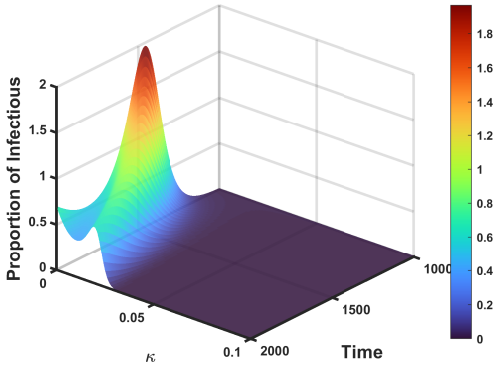
Now, we investigate the effects of the vaccine on the various vaccination rates on day 500 of the onset. The experiment results shown in Figs.6 (a) and (b) reveal that the number of infectious individuals increases at certain points for most vaccination rates, except for values close to 0.1. However, such high vaccination rates cannot be easily achieved in reality. This experiment helps demonstrate that vaccines alone are not sufficient to completely eradicate the disease. However, providing treatment helps drastically improve the situation, and thus,



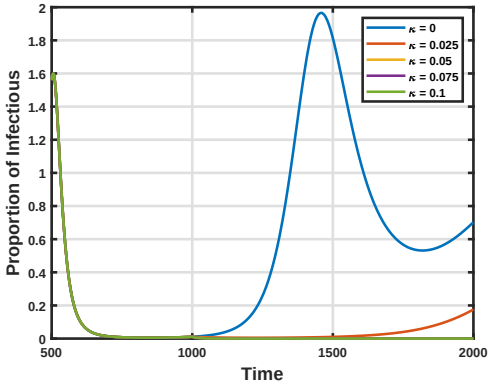
(a) Vaccination rate with only vaccine.



(b) Specific cases of vaccination rate with only vaccine.



(c) Treatment efficacy.



(d) Specific cases of treatment efficacy.

FIGURE 6. Sensitivity analysis of vaccination rate and treatment efficacy.

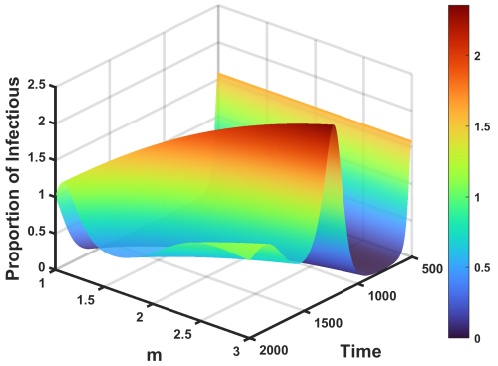
we moving closer to eradicating the disease, as shown in Figs.6 (c) and (d) (Table 2). The experimental results demonstrate that the disease is successfully eradicated for $\kappa \geq 0.037$. The basic reproduction number R_0 can be obtained easily, as formulated below:

$$R_0 = \frac{\beta(\theta + (1-\eta)\tau)}{(\gamma + \kappa)(\theta + \tau)}, \tag{2}$$

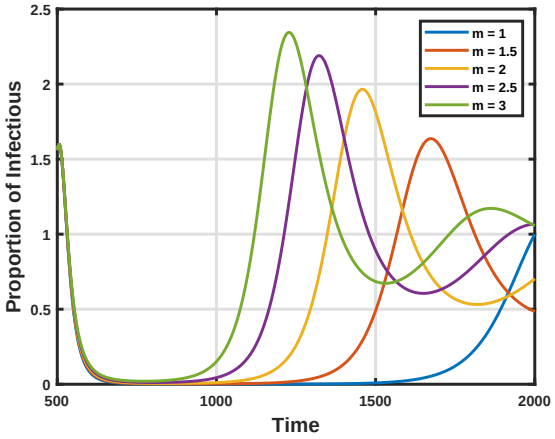
in the proposed model. In the setting for experiment, $R_0 = 0.1393/(0.1+\kappa) \leq 1$ for $\kappa \geq 0.0393$.

3.3.2. *Sensitivity study of the waning rates.* Now, we examine the effects of vaccines and treatments for various ratios ($m = \theta/\alpha$) of two waning rates experimentally, where we set the duration of natural infection–induced immunity to be higher than that of vaccination-induced immunity ([10,31]), that is, $m \geq 1$. We infer important information from Fig.7 (b). Assuming that the duration of immunity from natural infection and vaccination are the same ($m = 1$) for implementing only vaccine, the disease-free state is reached over time, as shown in Fig.7 (a). However, if there is a difference in the durations of two types of immunity, the infected individuals will continue to exist even after the vaccination.

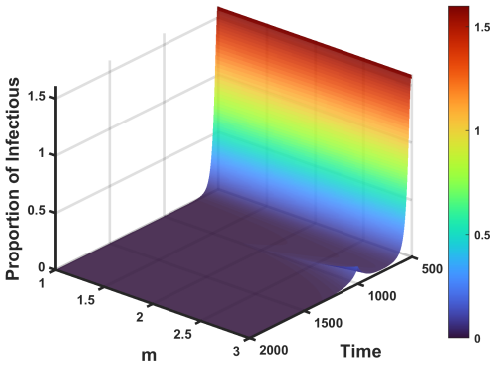
The results given in Figs.7 and 8 reveal that solely vaccination cannot eradicate the disease. When vaccines and treatments are used together, the disease is likely to disappear irrespective of the shorter duration of immunity from vaccination compared with that from infection. That is, after the treatment is developed, the epidemic equilibrium moves toward the disease-free equilibrium and stays on the time axis, where the number of infectious individuals decreases to zero.



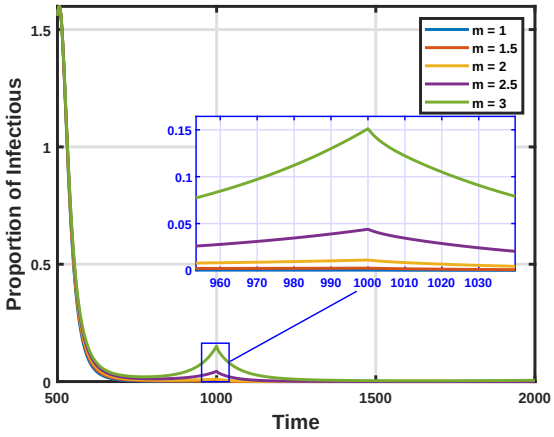
(a) Changing the ratio of two waning rates with only vaccine.



(b) Specific cases of the ratio of two waning rates with only vaccine.



(c) Various ratios of two waning rates with both vaccine and treatment.



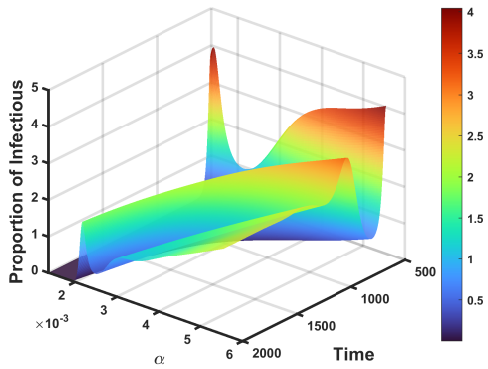
(d) Specific cases of the ratio of two waning rates with both vaccine and treatment.

FIGURE 7. Sensitivity analysis of various ratios ($m = \theta/\alpha$) of two waning rates.

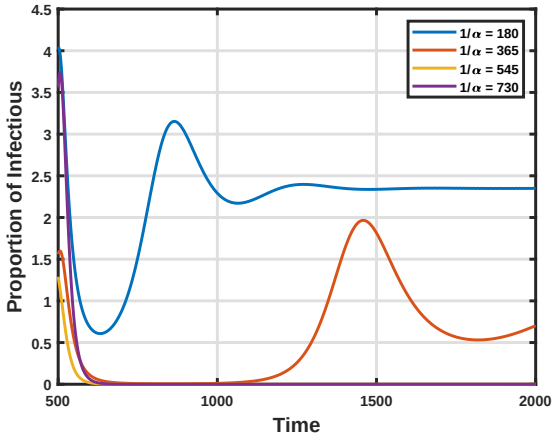
On the contrary, Fig.8 shows that vaccination and treatment together can lead to the disease-free state for any duration of natural immunity.

3.3.3. *Sensitivity study of the control effect according to the time of treatment implementation.* We have simulated the control effect according to the duration of treatment. The treatment period was selected from 800 to 1800 based on the simulation result of the vaccine effect estimated in Section 3.1—the epidemic equilibrium with 702(0.702%) infectious individuals after a local peak at $t = 1459$ with $1,966(1.966\%)$ infectious individuals. Irrespective of the timing of implementation, Fig.9 shows that the treatment exhibits the potential to eventually eradicate the infectious disease.

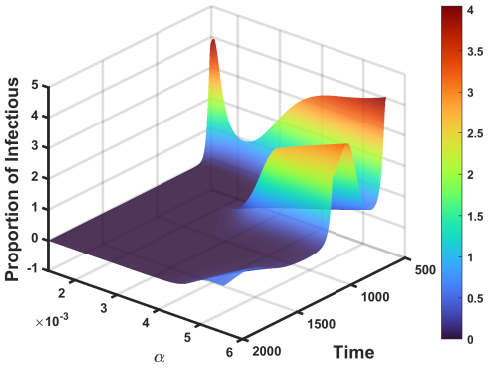
3.4. **Final epidemic size comparison.** This section examines and compares the final extent of the epidemic under different intervention strategies. We aim to demonstrate the significant impact of vaccination and treatment. The final epidemic size for each intervention strategy is presented in this section. Fig.10 shows the progression of the infectious population over time under three scenarios: no interventions, vaccination alone, and vaccination combined with



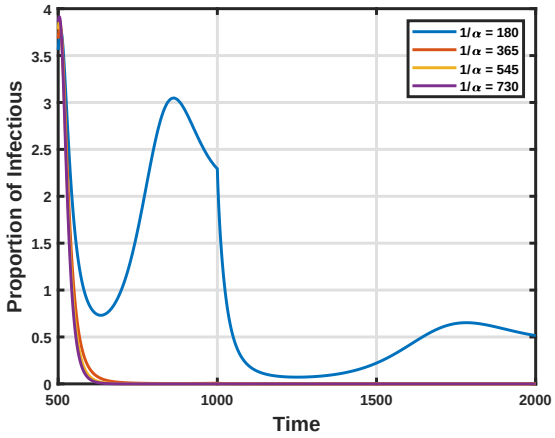
(a) Natural waning rate with only vaccine.



(b) Specific cases of natural waning rate with only vaccine.

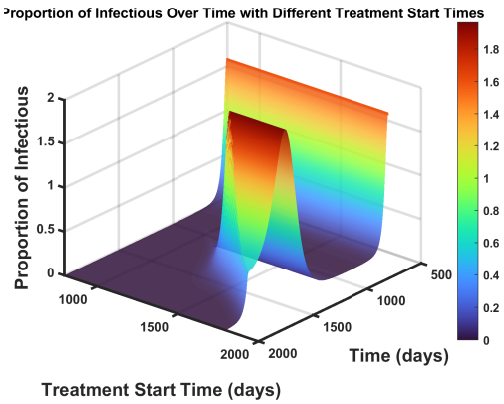


(c) Natural waning rate with both vaccine and treatment.

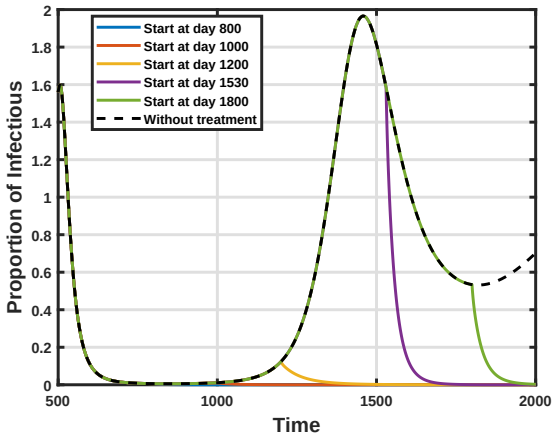


(d) Specific cases of natural waning rate with both vaccine and treatment.

FIGURE 8. Sensitivity analysis of natural waning rate.



(a) Treatment timing.



(b) Specific cases of treatment timing.

FIGURE 9. Sensitivity analysis of treatment timing.

TABLE 2. Comparison of disease-free equilibrium according to treatment and vaccination. The numbers in parentheses indicate the default values of the parameters.

Parameter		Only Vaccine	Both Vaccine and Treatment
τ (0.05)	0	∞	∞
	0.025	∞	∞
	0.05	∞	1988 days
	0.075	∞	1361 days
	0.1	∞	1230 days
κ (0.05)	0		∞
	0.025		∞
	0.05		1988 days
	0.075		1383 days
	0.1		1257 days
$\frac{1}{\alpha}$ (365)	180	∞	∞
	365	∞	1888 days
	545	∞	1111 days
	730	1022 days	990 days
m (2)	1	∞	1311 days
	1.5	∞	1508 days
	2	∞	1988 days
	2.5	∞	∞
	3	∞	∞
Treatment Timing (1000)	800		1315 days
	1000		1988 days
	1200		2832 days
	1530		2515 days
	1800		2868 days

treatment. For the first 500 days, no interventions are implemented, resulting in a steady increase in the number of infected individuals. After 500 days, three different scenarios are simulated:

- **No intervention:** The infectious population reaches a stable size of 2,104 individuals.
- **Vaccination only:** The infectious population is significantly reduced, but a resurgence is observed at $t = 1,458$, peaking at 1,966 individuals before gradually declining again.
- **Vaccination + Treatment:** The infectious population is effectively eradicated after day 944, and maintain disease free equilibrium.

The parameters and initial conditions employed for the initial 500 days remain unmodified, as delineated in Figs.4 and 5. In relation to the subsequent days, the simulations start by incorporating the final values from the previous run in order to achieve a seamless transition between phases. The final epidemic size, denoting the total infectious population at time t=2000, is presented.

The results unequivocally illustrate the advantage of integrating vaccination with treatment, as depicted in Fig.10. These findings provide evidence for the effectiveness of vaccination and treatment in extinguishing the epidemic. In contrast, vaccination alone considerably reduces the size of the epidemic but does not eliminate the infectious population entirely. The absence of intervention leads to a prolonged endemic state.

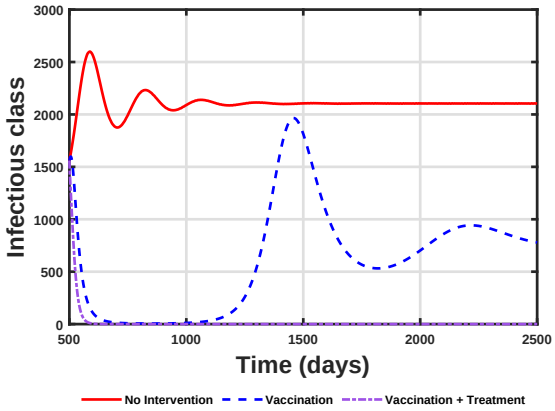


FIGURE 10. Final Epidemic Size Comparison: After 500 days without any interventions, three different scenarios are applied (no intervention, vaccination, and vaccination combined with treatment).

3.4.1. *Comparison of disease free equilibrium according to treatment and vaccine implementation.* Figs.6-9 show the simulation results of the sensitivity analysis for various parameters, and Table 2 presents the time required to reach the equilibrium point based on the values of the parameters. These results demonstrate that, when most cases are solely vaccinated, people continue to get infected, but along with treatment, individuals are no longer infected and then the epidemic is finally eradicated. Table 2 indicates that, based on the treatment implementation, the infectious disease is evidently nearing eradication; however, a considerable period of time is required to eradicate the disease after the treatment, implying that continuous infectious disease control and management are necessary even after the implementation of the therapeutic treatment. The treatment with $\kappa = 0.025$ does not lead to the disease-free equilibrium because the basic reproduction number R_0 is greater than one in this case from Eq. (2).

4. DISCUSSION

We employ the epidemic compartment model SVEIRS to evaluate the effect of vaccination and treatment for the control of the spread of infectious diseases. We suggest using different waning rates for infection- and vaccine-induced immunity in the model, as the assumption of the same ratios leads to the false conclusion that solely control by vaccination can eradicate infectious diseases, which we demonstrate via a simulation shown in Fig.1. Furthermore, significant evidence indicates that the rates of waning immunity for the two cases are different.

The experimental results obtained in this study suggest that the control of epidemic diseases through vaccination and treatment can end disease transmission, while control by vaccination alone does not eradicate the disease, as the epidemic state is maintained. Furthermore, we observe that the treatment development does not immediately eradicate the disease, and that it takes time for the disease to disappear completely due to infections by asymptomatic infected individuals.

To further support our analysis, we draw upon several recent studies that have explored related epidemic modeling techniques. The study conducted by the authors in [7] focused on investigating a two-strain SIV model with vaccination and latent delays. Their research provided valuable insights into the impact of delayed interventions, such as vaccination, on the dynamics of an epidemic. This finding emphasizes the significance of timely vaccination and treatment strategies, which is relevant to our analysis of COVID-19 interventions. The authors in [33] employed a cluster-based modeling approach that highlighted the necessity of targeted interventions in specific regions, particularly in densely populated or high-risk areas. This approach aligns with our work, as localized outbreaks were a major concern during the early stages of

the pandemic. In [37], a comprehensive stability and bifurcation analysis of epidemic models involving vaccination was conducted. We referenced this study to discuss how different vaccination and treatment rates can influence the stability of the disease-free equilibrium. Collectively, these studies enhance our understanding of how combined vaccination and treatment strategies can effectively control and ultimately eradicate infectious diseases.

5. Conclusion

In summary, the experimental results obtained through the proposed SVEIRS model suggest that the pandemic can be eradicated via both vaccination and treatment. The experiments have helped demonstrate that each step is time-consuming, i.e., to develop and distribute vaccines, develop treatments, and eradicate infectious diseases via treatment and vaccination. Thus, the development of vaccines and treatments does not indicate an immediate eradication of the disease. Therefore, the spread of the disease must be controlled until the disease has been completely eradicated. That is, if the vaccine alone controlled the disease, the disease does not end and shows epidemic equilibrium after a certain period of time. Thus, the development of post-vaccine treatments helps suppress the disease and eventually eradicates the disease.

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